Solution

PREBOARD EXAM- 2 2025-26

Class 12 - Mathematics

Section A

1.
   (c) 8
   Explanation:
   8

2. **(a)** Idempotent
   Explanation:
   clearly for given matrix $A^2 = A$
   Therefore idempotent

3. **(a)** 4|A|
   Explanation:
   A = $\begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, 2A = $\begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$
   $|2A| = 2^2 |A| = 4 |A|$

4.
   (c) (1 + sin 2x) y_1
   Explanation:
   $y = e^{\tan x}$
   $y_1 = \sec^2 x\, e^{\tan x}$
   $\Rightarrow \cos^2 x y_1 = e^{\tan x}$
   Again differentiating w.r.t. x we get
   $\cos^2 (x) .y_2 - 2 \cos x \sin x y_1 = \sec^2 x e^{\tan x}$
   $\Rightarrow \cos^2 (x).y_2 = y_1 \sin 2x + y_1.$

5.
   (d) $\frac{\pi}{2}$
   Explanation:
   Let's consider the first parallel vector to be $\vec{a} = a\hat{i} + b\hat{j} + c\hat{k}$ and second parallel vector be
   $\overrightarrow{\mathrm{b}} = (\mathrm{b} - \mathrm{c})\hat{\mathrm{i}} + (\mathrm{c} - \mathrm{a})\hat{\mathrm{j}} + (\mathrm{a} - \mathrm{b})\hat{\mathrm{k}}$
   For the angle, we can use the formula $\cos\alpha = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\times|\vec{b}|}$
   For that, we need to find the magnitude of these vectors
   $|\vec{a}| = \sqrt{a^2 + b^2 + (c)^2}$
   $= \sqrt{a^2 + b^2 + c^2}$
   $|\overrightarrow{\mathrm{b}}| = \sqrt{(\mathrm{b} - \mathrm{c})^2 + (\mathrm{c} - \mathrm{a})^2 + (\mathrm{a} - \mathrm{b})^2}$
   $= \sqrt{2\left(a^2 + b^2 + c^2 - ab - bc - ca\right)}$
   $\Rightarrow \cos\alpha = \frac{(a\hat{\imath} + b\hat{\jmath} + c\hat{k})\cdot((b-c)\hat{\imath} + (c-a)\hat{\jmath} + (a-b)\hat{k})}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$
   $\Rightarrow \cos\alpha = \frac{ab-ac+bc-ba+ca-cb}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$
   $\Rightarrow \cos\alpha = \frac{0}{\sqrt{2\left(a^2+b^2+c^2-ab-bc-ca\right)}\times\sqrt{a^2+b^2+c^2}}$

$\Rightarrow \alpha = \cos^{-1}(0)$
$\therefore \alpha = \frac{\pi}{2}$

6.

(d) $\frac{ax^2}{2} + bx$

Explanation:

$\frac{ax^2}{2} + bx$

7. **(a)** $x_1 = 2$, $x_2 = 6$, Z = 36

Explanation:

We need to maximize the function $z = 3x_1 + 5x_2$

First, we will convert the given inequations into equations, we obtain the following equations: $3x_1 + 2x_2 = 18$, $x_1 = 4$, $x_2 = 6$, $x_1 = 0$ and $x_2 = 0$

Region represented by $3x_1 + 2x_2 \leq 18$

The line $3x_1 + 2x_2 = 18$ meets the coordinate axes at A(6, 0) and B(0, 9) respectively. By joining these points we obtain the line $3x_1 +$

$2x_2 = 18$

Clearly (0, 0) satisfies the inequation $3x_1 + 2x_2 = 18$. So, the region in the plane which contain the origin represents the solution set of the inequation $3x_1 + 2x_2 = 18$

Region represented by $x_1 \leq 4$:

The line $x_1 = 4$ is the line that passes through C(4, 0) and is parallel to the Y axis. The region to the left of the line $x_1 = 4$ will satisfy the inequation $x_1 \leq 4$

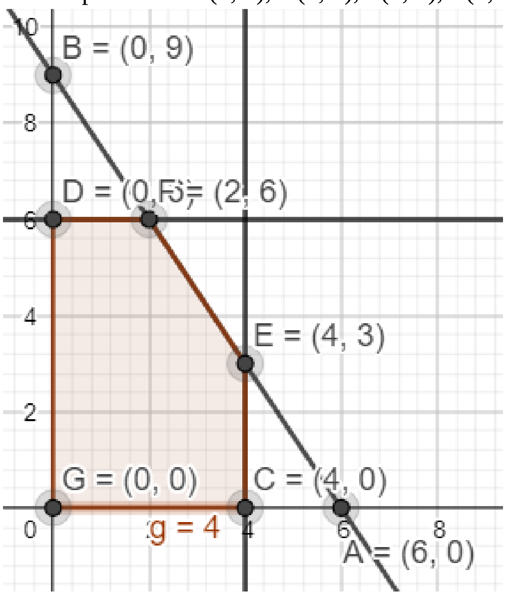
Region represented by $x_2 \leq 6$ The line $x_2 = 6$ is the line that passes through D(0, 6) and is parallel to the x axis. The region below the line $x_2 = 6$ will satisfy the inequation $x_2 \leq 6$.

Region represented by $x_1 \geq 0$ and $x_2 \geq 0$

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations $x_1 \geq$ oand $x_2 \geq 0$

The feasible region determined by the system of constraints, $3x_1 + 2x_2 \leq 18$, $x_1 \leq 4$, $x_2 \leq 6$, $x_1 \geq 0$, and $x_2 \geq 0$, are as follows :

Corner points are O(0, 0), D(0, 6), F(2, 6), E(4, 3) and C(4, 0).



The values of the objective function at these points are given in the following table

Points : Value of Z

O(0, 0) : 3(0) + 5(0) = 0

D(0, 6) : 3(0) + 5(6) = 30
F(2, 6) : 3(2) + 5(6) = 36
E(4, 3) : 3(4) + 5(3) = 27
C(4, 0) : 3(4) + 5(0) = 12
We see that the maximum value of the objective function Z is 36 which is at F(2, 6)

8.

(c) $\frac{-3}{2}$

Explanation:

It is given that: **If** $\vec{a}, \vec{b}, \vec{c}$ **are unit vectors such that** $\vec{a} + \vec{b} + \vec{c} = 0$,
then:

$(\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = \vec{0}.\vec{0}$

$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow 1 + 1 + 1 + 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = 0$

$\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\frac{3}{2}$

9.

(d) e^x log sec x + C

Explanation:

I = $\int$ e^x {f(x) + f'(x)}dx, where f(x) = log sec x

= e^x f(x) + C = e^x log sec x + C

10.

(d) [14]

Explanation:

[14]

11. **(a)** a function to be optimized

Explanation:

a function to be optimized

The objective function of a linear programming problem is either to be maximized or minimized i.e. objective function is to be optimized.

12. **(a)** 3, $\frac{27}{2}$

Explanation:

It is given that:

$\left(2\hat{i} + 6\hat{j} + 27\hat{k}\right) X \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right) = \overrightarrow{0}$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix} = \hat{i}\,(6\mu - 27\lambda) - \hat{j}\,(2\mu - 27) + \hat{k}\,(2\lambda - 6) = \overrightarrow{0}$, equating the coefficients of $\hat{i}, \hat{j}, \hat{k}$on both sides,we get

$(6\mu - 27\lambda) = 0, (2\mu - 27) = 0, (2\lambda - 6) = 0.$

solving,we get $\lambda = 3, \mu = \frac{27}{2}$

13.

(c) K^{n-1} Adj. A

Explanation:

Adj. (KA) = K^{n-1} Adj.A, where K is a scalar and A is a n $\times$ n matrix.

14.

(c) $\frac{14}{17}$

Explanation:

Here, $P(A \cap B) = \frac{7}{10}$ and P(B) = $\frac{17}{20}$,
$\therefore P(\frac{A}{B})$ = $\frac{P(A\cap B)}{P(B)} = \frac{7/10}{17/20} = \frac{14}{17}$

15. **(a)** 3, 2
Explanation:
We have, $\left(\frac{d^3y}{dx^3}\right)^2 - 3\frac{d^2y}{dx^2} + 2\left(\frac{dy}{dx}\right)^4 = y^4$
$\therefore$ Order = 3 and degree = 2

16. **(a)** $\frac{1}{2}(\vec{a} - \vec{b})$
Explanation:
Given parallelogram OACB such that $\overrightarrow{OC} = \vec{a}$
$\overrightarrow{AB} = \overrightarrow{b}$
$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$
$\overrightarrow{OB} = \vec{a} - \overrightarrow{OA}$ $(\because \overrightarrow{BC} = \overrightarrow{OA})$
$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
$\vec{a} - \overrightarrow{OA} = \overrightarrow{OA} + \vec{b}$
$\overrightarrow{OA} = \frac{\vec{a}-\vec{b}}{2}$

17.
(c) 3
Explanation:
f(x) = $\frac{1}{\log|x|}$
f(x) is not defined for x = 0, -1 ,1
$\therefore$ f(x) is not continuous at x = 0, -1, 1

18.
(c) $13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$
Explanation:
$13; \frac{12}{13}, \frac{4}{13}, \frac{3}{13}$
If a line makes angles α, β and γ with the axis, then $\cos^2\alpha + \cos^2\beta + \cos^2\gamma$ = 1 ....(i)
Let r be the length of the line segment. Then,
$r\cos\alpha = 12, r\cos\beta = 4, r\cos\gamma = 3$ ....(ii)
$\Rightarrow (r\cos\alpha)^2 + (r\cos\beta)^2 + (r\cos\gamma)^2$ = $12^2 + 4^2 + 3^2$
$\Rightarrow r^2\left(\cos^2\alpha + \cos^2\beta + \cos^2\gamma\right)$ = 169
$\Rightarrow r^2$ (1) = 169 [From (i)]
$\Rightarrow r = \sqrt{169}$
$\Rightarrow$ r = ± 13
$\Rightarrow$ r = 13 (since length cannot be negative)
Substituting r = 13 in (ii)
We get,
$\cos\alpha = \frac{12}{13}, \cos\beta = \frac{4}{13}, \cos\gamma = \frac{1}{13}$
Thus, the direction cosines of the line are
$\frac{12}{13}, \frac{4}{13}, \frac{1}{13}$

19.
(b) Both A and R are true but R is not the correct explanation of A.
Explanation:
For maximum value of $\frac{x}{y}$, we take maximum value of x and minimum value of y,
i.e., Maximum value of $\left(\frac{x}{y}\right) = \frac{10}{5} = 2$

Similarly, minimum value of $\left(\frac{x}{y}\right)=\frac{3}{15}=\frac{1}{5}$

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation:
Assertion: Since, greatest integer [x] gives only integer value.
But f(x) = [x] + x gives all real values and there is no repeated value of f(x) for any value of x.
Hence, f{x) is one-one and onto.

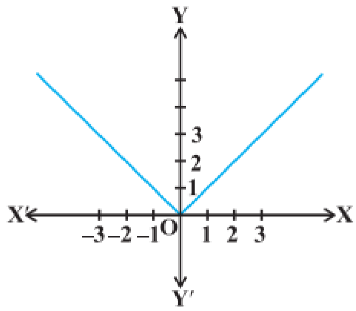
Section B

21. Let $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)=y$
$\Rightarrow \cos y=\frac{\sqrt{3}}{2}$
$\Rightarrow \cos y=\cos\frac{\pi}{6}$
$\Rightarrow y=\frac{\pi}{6}$
Since, the principal value branch of $\cos^{-1}$ is $[0,\ \pi]$.
Therefore, principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$.

OR

$\tan^{-1}\sqrt{3}-\sec^{-1}(-2)=\tan^{-1}\sqrt{3}-\left[\pi-\sec^{-1}2\right]$
$=\frac{\pi}{3}-\pi+\cos^{-1}\left(\frac{1}{2}\right)$
$=-\frac{2\pi}{3}+\frac{\pi}{3}=-\frac{\pi}{3}$

22. From the graph of the given function

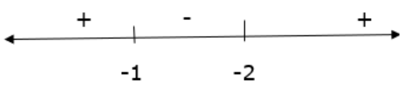


We know that f(x) $\geq$ 0, $\forall$ x $\in$ R
and f(x) = 0 if x = 0
Therefore, the function f has a minimum value '0' and the point of minimum value of f is x = 0. Also, the graph clearly shows that f has no maximum value in R and hence no point of the maximum value in R.

23. Let f(x) = x^4- 62 x^2 + ax + 9
$\Rightarrow f'(x)=4x^3-124x+a$
Since, f(x) attains its maximum value at x = 1 in the interval [0, 2], therefore $f'(1)=0$
$\therefore f'(1)=4-124+a=0$
$\Rightarrow a-120=0$
$\Rightarrow a=120$

OR

Given,f(x) = $2x^3$ + $9x^2$ + 12x + 15
f'(x) = $6x^2$ + 18x + 12
f'(x) = 6(x^2 + 3x + 2)
= 6(x + 2)(x + 1)



Therefore,function f(x) is decreasing for x $\in$ [-1, -2] and increasing in x $\in$ $(-\infty,-1)\cup(-2,\infty)$

24. $I=\int e^x(\sin x+\cos x)dx$
Now,
Let sin x = f(x) $\Rightarrow$ f'(x) = cos x
We know that,
$\int e^x\{f(x)+f'(x)\}dx=e^x f(x)+c$

Thus,

$\int e^x(\sin x+\cos x)dx = e^x\sin x + C$

25. Given: f(x) = -|x + 1| + 3

We know that -|x + 1| $\leq$ 0 for every x $\in$ R.

Therefore, f(x) = - |x+1| + 3 $\leq$ 3 for every x $\in$ R.

Hence, the maximum value of f is attained when |x+1| = 0

|x + 1| = 0

$\Rightarrow x=-1$

$\because$ Maximum value of f = f(-1) = -|-1 + 1| + 3 = 3

Hence, function f does not have a minimum value.

Section C

26. Let the given integral be,

I = $\int \frac{x+1}{x^2+x+3}dx$

Let x - 3 = $\lambda\frac{d}{dx}$ (x^2 + 2x - 4) + μ

= λ(2x + 2) + μ

$\Rightarrow$ x - 3 = (2λ)x + (2λ + μ)

Comparing the coefficients of like powers of x,

2λ = 1 $\Rightarrow$ $\lambda=\frac{1}{2}$

2λ + μ = -3 $\Rightarrow$ $2\left(\frac{1}{2}\right)$ + μ = -3

μ = - 4

So, $I=\int\frac{\frac{1}{2}(2x+2)-4}{x^2+2x-4}dx$

$I=\frac{1}{2}\int\frac{2x+2}{x^2+2x-4}dx-4\int\frac{1}{x^2+2x+(1)^2-(1)^2-4}dx$

$I=\frac{1}{2}\int\frac{2x+2}{x^2+2x-4}dx-4\int\frac{1}{(x+1)^2-(\sqrt{5})}dx$

$I=\frac{1}{2}$ log |x^2 + 2x - 4] - 4 $\times\frac{1}{2\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$ ...[Since, $\int\frac{1}{x^2-a^2}dx=\frac{1}{2a}\log|\frac{x-a}{x+a}$ | + c]

$I=\frac{1}{2}\log\left|x^2+2x-4\right|-\frac{2}{\sqrt{5}}\log\left|\frac{x+1-\sqrt{5}}{x+1+\sqrt{5}}\right|+c$

27. A white ball and a red ball can be drawn in three mutually exclusive ways:

i. Selecting bag I and then drawing a white and a red ball from it
ii. Selecting bag II and then drawing a white and a red ball from it
iii. Selecting bag III and then drawing a white and a red ball from it

Consider the following events:

E_1 = Selecting bag I

E_2 = Selecting bag II

E_3 = Selecting bag II

A = Drawing a white and a red ball

It is given that one of the bags is selected randomly.

$\therefore P(E_1)=\frac{1}{3}$

P(E_2) = $\frac{1}{3}$

P(E_3) = $\frac{1}{3}$

Now,

$P\left(\frac{A}{E_1}\right)=\frac{{}^1C_1\times{}^3C_1}{{}^6C_2}=\frac{3}{15}$

$P\left(\frac{A}{E_2}\right)=\frac{{}^2C_1\times{}^1C_1}{{}^4C_2}=\frac{2}{6}$

$P\left(\frac{A}{E_3}\right)=\frac{{}^4C_1\times{}^3C_1}{{}^{12}C_2}=\frac{12}{66}$

Using the law of total probability, we get

Required probability = P(A) = P(E_1)P($\frac{A}{E_1}$) + P(E_2)P($\frac{A}{E_2}$) + P(E_3)P($\frac{A}{E_3}$)

$=\frac{1}{3}\times\frac{3}{15}+\frac{1}{3}\times\frac{2}{6}+\frac{1}{3}\times\frac{12}{66}$

$=\frac{1}{15}+\frac{1}{9}+\frac{2}{33}$

$=\frac{33+55+30}{495}=\frac{118}{495}$

28. We can write the given function as, $\begin{cases} 2\tan^{-1}x & , \quad \text{if } -1<x<1 \\ -\pi+2\tan^{-1}x & , \quad \text{if } x>1 \\ \pi+2\tan^{-1}x & , \quad \text{if } x<-1 \end{cases}$

$\therefore \quad I=\int_0^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$

$\Rightarrow \quad I=\int_0^1 \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx+\int_1^{\sqrt{3}} \tan^{-1}\left(\frac{2x}{1-x^2}\right)dx$ [Using additive property]

$\Rightarrow \quad I=\int_0^1 2\tan^{-1}x dx+\int_1^{\sqrt{3}}\left(-\pi+2\tan^{-1}x\right)dx$

$\Rightarrow \quad I=\int_0^1 2\tan^{-1}x dx+\int_1^{\sqrt{3}}-\pi dx+\int_1^{\sqrt{3}} 2\tan^{-1}x dx$

$\Rightarrow \quad I=\left\{\int_0^1 2\tan^{-1}x dx+\int_1^{\sqrt{3}} 2\tan^{-1}x dx\right\}-\pi\int_1^{\sqrt{3}} 1\cdot dx$

$\Rightarrow \quad I=2\int_0^{\sqrt{3}} \tan^{-1}x dx-\pi\int_1^{\sqrt{3}} 1\cdot dx$

$\Rightarrow \quad I=2\left[\left\{x\tan^{-1}x\right]_0^{\sqrt{3}}-\int_0^{\sqrt{3}}\frac{x}{1+x^2}dx\right]-\pi[x]_1^{\sqrt{3}}$

$\Rightarrow \quad I=2\left[\left\{\sqrt{3}\tan^{-1}\sqrt{3}-0\right\}-\frac{1}{2}\left[\log\left(1+x^2\right)\right]_0^{\sqrt{3}}\right]-\pi(\sqrt{3}-1)$

$\Rightarrow \quad I=2\left[\left\{\sqrt{3}\tan^{-1}\sqrt{3}-0\right\}-\frac{1}{2}\left[\log\left(1+x^2\right)\right]_0^{\sqrt{3}}\right]-\pi(\sqrt{3}-1)$

$\Rightarrow \quad I=\frac{2\pi}{3}\sqrt{3}-\log 4-\pi(\sqrt{3}-1)=\pi\left(1-\frac{1}{\sqrt{3}}\right)-\log 4$

OR

Let x = $\lambda\frac{d}{dx}$ (x^2 + x +1) + μ. Then, x = λ(2x + 1) + μ

Comparing the coefficients of like powers of x, we get

1 = 2λ and λ + μ = 0 $\Rightarrow \lambda=\frac{1}{2}$ and μ = -λ = $-\frac{1}{2}$

$\therefore \quad I=\int\frac{x}{x^2+x+1}dx$

$\Rightarrow \quad I=\int\frac{1/2(2x+1)-1/2}{x^2+x+1}dx$

by using the values of λ , and μ ,

$\Rightarrow \quad I=\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\frac{1}{2}\int\frac{1}{x^2+x+1}dx$

$\Rightarrow \quad I=\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\frac{1}{2}\int\frac{1}{\left(x^2+x+\frac{1}{4}\right)+\frac{3}{4}}dx$

$\Rightarrow \quad I=\frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx-\frac{1}{2}\int\frac{1}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}dx$

$\Rightarrow \quad I=\frac{1}{2}\log|x^2+x+1|-\frac{1}{2}\times\frac{1}{(\sqrt{3/2})}\tan^{-1}\left(\frac{x+1/2}{\sqrt{3/2}}\right)+C$

$\Rightarrow \quad I=\frac{1}{2}\log|x^2+x+1|-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

29. The given differential equation is,

$\frac{dy}{dx}$ + 2y = sin x

It is a linear differential equation. Comparing the equation with,

$\frac{dy}{dx}$ + Py = Q

P = 2, Q = sin x

I.F. $=e^{\int pdx}$

$=e^{\int dx}$

= e^{2x}

Solution of the equation is given by,

y $\times$ (I.F.) = $\int$ Q $\times$ (I.F.) dx + c

y(e^{2x}) = $\int$ sinx $\times$ (e^{2x}) dx + c

ye^{2x} = $\frac{e^{2z}}{5}$ (2 sin x - cos x) + c

OR

We have differential equation,

$x\left(x^2-1\right)\frac{dy}{dx}=1$

$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{x\left(x^2-1\right)}$

$\Rightarrow \quad \frac{dy}{dx}=\frac{1}{x(x-1)(x+1)}$ $\left[\because a^2-b^2=(a-b)(a+b)\right]$

$\Rightarrow \quad dy=\frac{dx}{x(x-1)(x+1)}$

Therefore, on integrating both sides, we get

$\int dy=\int \frac{dx}{x(x-1)(x+1)}$

$\Rightarrow \quad y=I+K$ ...(i)

where, $I=\int \frac{dx}{x(x-1)(x+1)}$

By using partial fraction method,

let $\frac{1}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$

$\Rightarrow$ 1 = A(x - 1)(x + 1) + Bx(x + 1) + Cx(x - 1)

$\Rightarrow$ 1 = A(x^2 - 1) + B(x^2 + x) + C(x^2 - x)

Therefore, on comparing the coefficients of x^2, x and constant terms from both sides, we get

A + B + C = 0

B - C = 0

and -A = 1

$\Rightarrow$ A = -1

Therefore, on solving above equations, we get

A = -1, $B=\frac{1}{2}$ and $C=\frac{1}{2}$,

then $\frac{1}{x(x-1)(x+1)}=\frac{-1}{x}+\frac{1/2}{x-1}+\frac{1/2}{x+1}$

Therefore,on integrating both sides w.r.t. x, we get

$I=\int \frac{1}{x(x-1)(x+1)} dx=\int \frac{-1}{x} dx+\frac{1}{2} \int \frac{dx}{x-1}+\frac{1}{2} \int \frac{dx}{x+1}$

$\Rightarrow \quad I=-\log |x|+\frac{1}{2} \log |x-1|+\frac{1}{2} \log |x+1|$

Therefore,on putting the value of I in Eq. (i), we get

$y=-\log |x|+\frac{1}{2} \log |x-1|+\frac{1}{2} \log |x+1|+K$ ....(ii)

Also, we have, y = 0, when x = 2

Therefore,on putting y = 0 and x = 2 in Eq. (ii), we get

$0=-\log 2+\frac{1}{2} \log 1+\frac{1}{2} \log 3+K$

$\Rightarrow \quad K=\log 2-\frac{1}{2} \log 1-\frac{1}{2} \log 3$

$\Rightarrow \quad K=\log 2-\log \sqrt{3} \qquad [\because \log 1=0]$

$\Rightarrow \quad K=\log \frac{2}{\sqrt{3}}$

Therefore,on putting the value of K in Eq. (i), we get

$y=-\log |x|+\frac{1}{2} \log |x-1|+\frac{1}{2} \log |x+1|+\log \frac{2}{\sqrt{3}}$

which is the required solution.

30. First, we will convert the given inequations into equations, we obtain the following equations:

x + y = 8, x + 4 y = 12, x = 3, y = 2 and solving we get values are as follows:

The region represented by x + y $\geq$ 8: The line x + y = 8 meets the coordinate axes at A(8,0) and B(0,8) respectively. By joining these points we obtain the line x + y = 8 Clearly (0,0) does not satisfies the inequation x + y $\geq$ 8 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation x + y $\geq$ 8

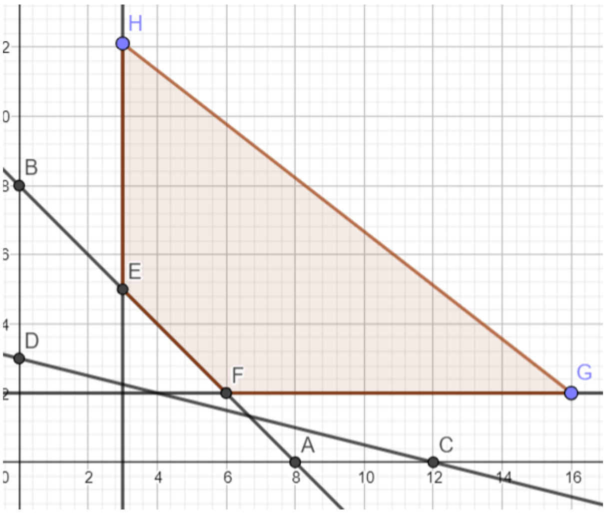
The region represented by x + 4 y $\geq$ 12 :

The line x + 4y = 12 meets the coordinate axes at C(12,0) and D(0,3) respectively. By joining these points we obtain the line x + 4y = 12.

Clearly (0,0) satisfies the inequation x + 4y $\geq$ 12 . So, the region in x y plane which contains the origin represents the solution set of the inequation x + 4y $\geq$ 12

The line x = 3 is the line that passes through the point (3,0) and is parallel to Y axis x $\geq$ 3 is the region to the right of the line x = 3

The line y = 2 is the line that passes through the point (0,12) and is parallel to X axis.y $\geq$ 2 is the region above the line y = 2



The corner points of the feasible region are E(3,5) and F(6,2)

The values of Z at these corner points are as follows.

Corner point	Z = 2x + 4y
E(3, 5)	$2 \times 3 + 4 \times 5 = 26$
F(6, 2)	$2 \times 6 + 4 \times 2 = 20$

Therefore, the minimum value of objective function Z is 20 at point F(6,2). Hence, x = 6 and y = 2 is the optimal solution of the given LPP. Thus, the optimal value of objective function z is 20.

OR

First, we will convert the given inequations into equations, we obtain the following equations:

5x + y = 10, x + y = 6, x + 4y = 12, x = 0 and y = 0

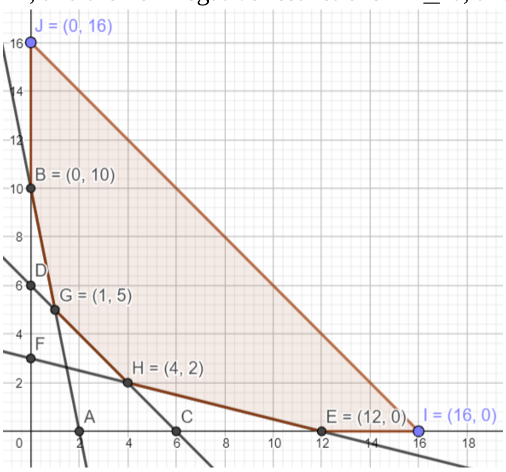
Region represented by 5x + y $\geq$ 10 :

The line 5x + y = 10 meets the coordinate axes at A(2,0) and B(0,10) respectively. By joining these points we obtain the line 5x + y = 10. Clearly (0,0) does not satisfies the inequation 5x + y $\geq$ 10 . So, the region in x y plane which does not contain the origin represents the solution set of the inequation 5x + y $\geq$ 10 Region represented by x + y $\geq$ 6 :

The line x + y = 6 meets the coordinate axes at C(6,0) and D(0,6) respectively. By joining these points we obtain the line 2 x + 3 y = 30. Clearly (0,0) does not satisfies the inequation x + y $\geq$ 6 . So, the region which does not contain the origin represents the solution set of the inequation 2 x + 3 y $\geq$ 30 Region represented by x + 4y $\geq$ 12 The line x + 4y = 12 meets the coordinate axes at E(12,0) and F(0,3) respectively. By joining these points we obtain the line

x + 4y = 12 . Clearly (0,0) does not satisfies the inequation x + 4y $\geq$ 12 . So, the region which does not contain the origin represents the solution set of the inequation x + 4y $\geq$ 12 Region represented by x $\geq$ 0 and y $\geq$ 0 :

since, every point in the first quadrant satisfies these inequations. So, the first quadrant is the region represented by the inequations x $\geq$ 0, and y $\geq$ 0. The feasible region determined by subject to the constraints are 5x + y $\geq$ 10, x + y $\geq$ 6, x + 4y $\geq$ 12, and the non-negative restrictions xx $\geq$ 0, and y $\geq$ 0, are as follows.



The corner points of the feasible region are B(0,10), G(1,5), H(4,2) and E(12,0)

The values of objective function Z at these corner points are as follows.

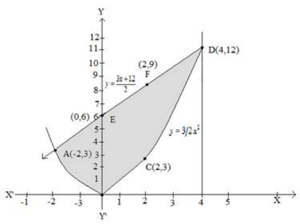
Corner point Z = 3x + 2y

B(0, 10) : $3 \times 0 + 3 \times 10 = 30$

G(1, 5) : $3 \times 1 + 2 \times 5 = 13$
H(4, 2) : $3 \times 4 + 2 \times 2 = 16$
B(12, 0) : $3 \times 12 + 2 \times 0 = 36$
Therefore, the minimum value of Z is 13 at the point G(1,5) . Hence, x = 1 and y = 5 is the optimal solution of the given LPP.
The optimal value of objective function Z is 13.

31. Given, x = a$\left(\cos t + \log\tan\frac{t}{2}\right)$ ....(i)
and y = a sin t..........(ii)
Therefore, on differentiating both sides w.r.t t, we get,
$\frac{dx}{dt} = a\left[\frac{d}{dt}(\cos t) + \frac{d}{dt}\log\tan\frac{t}{2}\right]$
$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\frac{d}{dt}\left(\tan\frac{t}{2}\right)\right]$ [by using chain rule of derivative]
$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}}\cdot\sec^2\frac{t}{2}\frac{d}{dt}\left(\frac{t}{2}\right)\right]$
$= a\left[-\sin t + \frac{1}{\tan\frac{t}{2}} \times \sec^2\frac{t}{2} \times \frac{1}{2}\right]$
$= a\left[-\sin t + \frac{1}{\frac{\sin t/2}{\cos t/2}} \times \frac{1}{\cos^2\frac{t}{2}} \times \frac{1}{2}\right]$
$= a\left[-\sin t + \frac{1}{2\sin\frac{t}{2}\cdot\cos\frac{t}{2}}\right]$
$= a\left[-\sin t + \frac{1}{\sin t}\right]$ $[\because \sin 2\theta = 2\sin\theta\cos\theta]$
$= a\left(\frac{1-\sin^2 t}{\sin t}\right)$
$\Rightarrow \quad \frac{dx}{dt} = a\left(\frac{\cos^2 t}{\sin t}\right)$ $[\because 1 - \sin^2\theta = \cos^2\theta]$ ....(iii)
Again,on differentiating both sides of (ii) w.r.t t, we get,
$\frac{dy}{dt}$ = a cos t...............(iv)
Therefore, $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{a\cos t}{a\left(\frac{\cos^2 t}{\sin t}\right)}$ [from Eqs(iii) and (iv)]
$= \frac{a\cos t}{a\cos^2 t} \times \sin t$ = tan t
Therefore,on differentiating both sides of above equation w.r.t x, we get,
$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(\tan t)$
$= \frac{d}{dt}(\tan t)\frac{dt}{dx}\left[\because \frac{d}{dx}f(t) = \frac{d}{dt}f(t)\cdot\frac{dt}{dx}\right]$
$\Rightarrow \quad \frac{d^2y}{dx^2} = \sec^2 t \times \frac{\sin t}{a\cos^2 t}$ [From Eq.(iii)]
$\Rightarrow \quad \frac{d^2y}{dx^2} = \frac{\sin t\sec^4 t}{a}$
Therefore, on putting t = $\frac{\pi}{3}$,we get,
$\left[\frac{d^2y}{dx^2}\right]_{t=\frac{\pi}{3}} = \frac{\sin\frac{\pi}{3}\times\sec^4\frac{\pi}{3}}{a} = \frac{\frac{\sqrt{3}}{2}\times(2)^4}{a}$
$= \frac{8\sqrt{3}}{a}$

Section D

32. 

$4y = 3x^2$ ......(1)
2y = 3x + 12......(2)
From (2), $y = \frac{3x+12}{2}$
Using this value of y in (1), we get,

x^2 - 6x - 8 = 0

⇒ (x + 2) (x - 4) = 0

⇒ x = -2, 4

From (2),

When, x = -2, y = 3

When, x = 4, y = 12

Thus, points of intersection are, (-2, 3) and (4, 12).

Area $= \int_{-2}^{4} \frac{3x+12}{2} dx - \int_{-2}^{4} \frac{3}{4} x^2 dx$

$= \frac{1}{2} [\frac{3x^2}{2} + 12x]^4{}_{-2} - \frac{3}{4} [\frac{x^3}{3}]^4_{-2}$

$\frac{1}{2} [(24 + 48) - (6 - 24)] - \frac{1}{4} [64 - (-8)]$

= 45 - 18 = 27 sq units.

33. We observe the following properties of relation R.

Reflexivity: For any a ∈N

a - a = 0 = 0 × n

⇒ a - a is divisible by n

⇒ (a, a) ∈ R

Thus, (a, a) ∈ for all a ∈ Z. So, R is reflexive on Z

Symmetry: Let (a, b) ∈R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ (a - b) = np for some p ∈ Z

⇒ b - a = n (-p)

⇒ b - a is divisible by n $[\because p \in Z \Rightarrow -p \in Z]$

⇒ (b, a) ∈ R

Thus, (a, b) ∈ R ⇒ (b, a) ∈R for all a, b ∈ Z.

So, R is symmetric on Z.

Transitivity: Let a, b, c ∈ Z such that (a, b) ∈R and (b, c) ∈ R. Then,

(a, b) ∈ R

⇒ (a - b) is divisible by n

⇒ a - b = np for some p ∈ Z

and, (b, c) ∈ R

⇒ (b - c) is divisible by n

⇒ b - c = nq for some q ∈ Z

∴ (a, b) ∈ R and (b, c) ∈ R

⇒ a - b = np and b - c = nq

⇒ (a - b) + (b - c) = np + nq

⇒ a - c = n (p + q)

⇒ a - c is divisible by n $[\because p, q \in Z \Rightarrow p + q \in Z]$

⇒ (a, c) ∈ R

Thus, (a, b) ∈ R and (b, c) ∈ R ⇒(a, c) ∈ R for all a, b, c ∈ Z.

OR

We observe the following properties of f.

Injectivity: Let x, y ∈ R_0 such that f(x) = f(y). Then,

$f(x) = f(y) \Rightarrow \frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f : $R_0 \rightarrow R_0$ is one-one.

Surjectivity: Let y be an arbitrary element of R_0 (co-domain) such that f(x) = y. Then,

$f(x) = y \Rightarrow \frac{1}{x} = y \Rightarrow x = \frac{1}{y}$

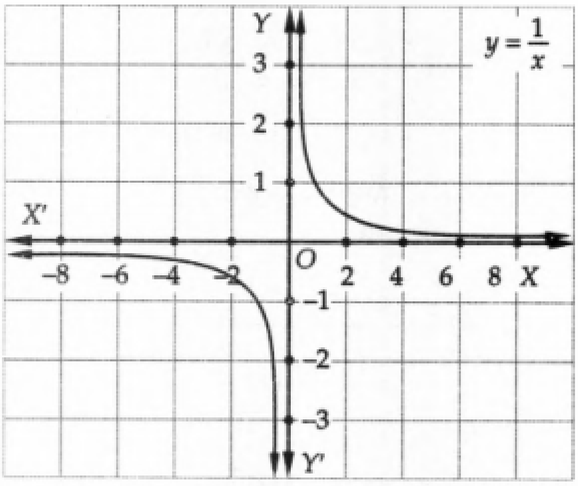
Clearly, $x = \frac{1}{y} \in R_0$ (domain) for all y ∈ R_0 (co-domain).

Thus, for each y ∈ R_0 (co-domain) there exits $x = \frac{1}{y} \in R_0$ (domain) such that $f(x) = \frac{1}{x} = y$

So, f : $R_0 \rightarrow R_0$ is onto.

Hence, f: $R_0 \rightarrow R_0$ is one-one onto.

This is also evident from the graph of f(x) as shown in fig.



Let us now consider f : N → R_0 given by $f(x) = \frac{1}{x}$

For any x, y ∈ N, we find that

f(x) = f(y) ⇒ $\frac{1}{x} = \frac{1}{y} \Rightarrow x = y$

So, f: N → R_0 is one-one.

We find that $\frac{2}{3}, \frac{3}{5}$ etc. in co-domain R_0 do not have their pre-image in domain N. So, f: N →R_0 is not onto.

Thus, f: N →R_0 is one-one but not onto.

34. Given: Matrix $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

$\therefore |A| = \begin{vmatrix} 3 & 7 \\ 2 & 5 \end{vmatrix} = 15 - 14 = 1 \neq 0$

$\therefore A^{-1} = \frac{1}{|A|} adj. A = \frac{1}{1}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

Matrix $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$\therefore |B| = \begin{vmatrix} 6 & 8 \\ 7 & 9 \end{vmatrix} = 54 - 56 = -2 \neq 0$

$\therefore B^{-1} = \frac{1}{|B|} adj. B = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

Now $AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix} = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix} = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$\therefore |AB| = \begin{vmatrix} 67 & 87 \\ 47 & 61 \end{vmatrix}$ = 67(61) - 87(47) = 4087 - 4089 $= -2 \neq 0$

Now L.H.S. = $(AB)^{-1} = \frac{1}{|AB|} adj. (AB) = \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ….(i)

R.H.S. = $B^{-1}A^{-1} = \frac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}\begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 45+16 & -63-24 \\ -35-12 & 49+18 \end{bmatrix}$

$= \frac{1}{-2}\begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$ ….(ii)

∴ From eq. (i) and (ii), we get

L.H.S. = R.H.S.

$\Rightarrow (AB)^{-1} = B^{-1}A^{-1}$

35. Here,it is given that

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

$\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

Here,

$\overrightarrow{a_1} = i + 2\hat{j} + 3\hat{k}$

$\overrightarrow{b_1} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\overrightarrow{a_2} = 4\hat{i} + \hat{j}$

$\vec{b_2} = 5\hat{i} + 2\hat{j} + \hat{k}$

Thus,

$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 5 & 2 & 1 \end{vmatrix}$

$= \hat{i}(3 - 8) - \hat{j}(2 - 20) + \hat{k}(4 - 15)$

$\therefore \vec{b_1} \times \vec{b_2} = -5\hat{i} + 18\hat{j} - 11\hat{k}$

$\therefore |\vec{b_1} \times \vec{b_2}| = \sqrt{(-5)^2 + 18^2 + (-11)^2}$

$= \sqrt{25 + 324 + 121}$

$= \sqrt{470}$

$\vec{a_2} - \vec{a_1} = (4 - 1)\hat{i} + (1 - 2)\hat{j} + (0 - 3)\hat{k}$

$\therefore \vec{a_2} - \vec{a_1} = 3\hat{i} - \hat{j} - 3\hat{k}$

Now,we have

$(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) = (-5\hat{i} + 18\hat{j} - 11\hat{k}) \cdot (3\hat{i} - \hat{j} - 3\hat{k})$

$= ((-5) \times 3) + (18 \times (-1)) + ((-11) \times (-3))$

= -15 - 18 + 33

= 0

Thus, the distance between the given lines is

$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right|$

$\therefore d = \left| \frac{0}{\sqrt{470}} \right|$

$\therefore$ d = 0 units

As d = 0

Thus, the given lines intersect each other.

Now, to find a point of intersection, let us convert given vector equations into Cartesian equations.

For that putting $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ in given equations,

$\Rightarrow \vec{L}_1 : x\hat{i} + y\hat{j} + z\hat{k} = (i + 2j + 3\hat{k}) + \lambda(2i + 3\hat{j} + 4\hat{k})$

$\Rightarrow \vec{L}_2 : x\hat{i} + y\hat{j} + z\hat{k} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + 2\hat{j} + \hat{k})$

$\Rightarrow \vec{L}_1 : (x - 1)\hat{i} + (y - 2)\hat{j} + (z - 3)\hat{k} = 2\lambda\hat{i} + 3\lambda\hat{j} + 4\lambda\hat{k}$

$\Rightarrow \vec{L}_2 : (x - 4)\hat{i} + (y - 1)\hat{j} + (z - 0)\hat{k} = 5\mu\hat{i} + 2\mu\hat{j} + \mu\hat{k}$

$\Rightarrow \vec{L}_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$\therefore \vec{L}_2 : \frac{x-4}{5} = \frac{y-1}{2} = \frac{z-0}{1} = \mu$

General point on L1 is

$x_1 = 2\lambda + 1$, $y_1 = 3\lambda + 2$, $z_1 = 4\lambda + 3$

Suppose, $P(x_1, y_1, z_1)$ be point of intersection of two given lines.

Thus, point P satisfies the equation of line $\vec{L}_2$.

$\Rightarrow \frac{2\lambda+1-4}{5} = \frac{3\lambda+2-1}{2} = \frac{4\lambda+3-0}{1}$

$\therefore \frac{2\lambda-3}{5} = \frac{3\lambda+1}{2}$

$\Rightarrow 4\lambda - 6 = 15\lambda + 5$

$\Rightarrow 11\lambda = -11$

$\Rightarrow \lambda = -1$

Thus, $x_1 = 2(-1) + 1$, $y_1 = 3(-1) + 2$, $z_1 = 4(-1) + 3$

$\Rightarrow x_1 = -1$, $y_1 = -1$, $z_1 = -1$

Therefore, point of intersection of given lines is (-1, -1, -1).

OR

Here,it is given equations of lines:

$L_1 : \frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$

$L_2 : \frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$

Direction ratios of L_1 and L_2 are (3, -1, 1) and (-3, 2, 4) respectively.

Suppose general point on line L_1 is P = (x_1, y_1, z_1)

$x_1 = 3s + 6$, $y_1 = -s + 7$, $z_1 = s + 4$

and suppose general point on line L_2 is Q = (x_2, y_2, z_2)

$x_2 = -3t$, $y_2 = 2t - 9$, $z_2 = 4t + 2$

$\therefore \overrightarrow{\mathrm{PQ}} = (\mathrm{x}_2 - \mathrm{x}_1)\,\hat{\mathrm{i}} + (\mathrm{y}_2 - \mathrm{y}_1)\,\hat{\mathrm{j}} + (z_2 - z_1)\,\hat{k}$
$= (-3t - 3s - 6)i + (2t - 9 + s - 7)\hat{j} + (4t + 2 - s - 4)\hat{k}$
$\therefore \overrightarrow{PQ} = (-3t - 3s - 6)\,\hat{\imath} + (2t + s - 16)\} + (4t - s - 2)\hat{k}$
Direction ratios of $\overrightarrow{\mathrm{PQ}}$ are ((-3t - 3s - 6), (2t + s - 16), (4t - s - 2))
PQ will be the shortest distance if it is perpendicular to both the given lines
Thus, by the condition of perpendicularity,
$\implies$ 3(-3t - 3s - 60) - 1(2t + s - 16) + 1(4t - s - 2) = 0 and
$\implies$ -3(-3t - 3s - 6) + 2(2t + s - 16) + 4(4t - s - 2) = 0
⇒ -9t - 9s - 18 - 2t - s + 16 + 4t - s - 2 = 0 and
9t + 9s + 18 + 4t + 2s + - 32 + 16t - 4s - 8 = 0
⇒ -7t - 11s = 4 and
29t + 7s = -22
Solving above two equations,the we obtain
t = 1 and s = -1
therefore
P = (3, 8, 3) and Q = (-3, -7, 6)
Now, distance between points P and Q is
$d = \sqrt{(3+3)^2 + (8+7)^2 + (3-6)^2}$
$= \sqrt{(6)^2 + (15)^2 + (-3)^2}$
$= \sqrt{36 + 225 + 9}$
$= \sqrt{270}$
$= 3\sqrt{30}$
Thus, the shortest distance between two given lines is
$d = 3\sqrt{30}$ units
Now, the equation of the line passing through points P and Q is
$\frac{x-x_1}{x_1-x_2} = \frac{y-y_1}{y_1-y_2} = \frac{z-z_1}{z_1-z_2}$
$\therefore \frac{x-3}{3+3} = \frac{y-8}{8+7} = \frac{z-3}{3-6}$
$\therefore \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{-3}$
$\therefore \frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$ thus, the equation of the line of the shortest distance between two given lines is
$\frac{x-3}{2} = \frac{y-8}{5} = \frac{z-3}{-1}$

Section E

36. i. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change
$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$
$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$
$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$
$= \frac{\frac{4}{7}\times 0.3}{\frac{4}{7}\times 0.3 + \frac{1}{7}\times 0.8 + \frac{2}{7}\times 0.5} = \frac{\frac{1.2}{7}}{\frac{1.2}{7} + \frac{0.8}{7} + \frac{1}{7}} = \frac{\frac{1.2}{7}}{\frac{3}{7}}$
$= \frac{1.2}{3} = \frac{12}{30} = \frac{2}{5}$

ii. Let E_1: Ajay(A) is selected, E_2: Ramesh(B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change
$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$
$P(A/E_1) = 0.3$, $P(A/E_2) = 0.8$, $P(A/E_3) = 0.5$

P(E_2/A) = $\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

= $\frac{\frac{1}{7}\times 0.8}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ = $\frac{\frac{0.8}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{\frac{0.8}{7}}{\frac{3}{7}}$

= $\frac{0.8}{3} = \frac{8}{30} = \frac{4}{15}$

iii. Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

P(A/E_1) = 0.3, P(A/E_2) = 0.8, P(A/E_3) = 0.5

P(E_3/A) = $\frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\cdot P(A/E_1)+P(E_2)P(A/E_2)+P(E_3)P(A/E_3)}$

= $\frac{\frac{2}{7}\times 0.5}{\frac{4}{7}\times 0.3+\frac{1}{7}\times 0.8+\frac{2}{7}\times 0.5}$ = $\frac{\frac{1}{7}}{\frac{1.2}{7}+\frac{0.8}{7}+\frac{1}{7}} = \frac{1}{3}$

OR

Let E_1: Ajay (A) is selected, E_2: Ramesh (B) is selected, E_3: Ravi (C) is selected

Let A be the event of making a change

$P(E_1) = \frac{4}{7},\ P(E_2) = \frac{1}{7},\ P(E_3) = \frac{2}{7}$

P(A/E_1) = 0.3, P(A/E_2) = 0.8, P(A/E_3) = 0.5

Ramesh or Ravi

$\Rightarrow$ P(E_2/A) + P(E_3/A) = $\frac{4}{15} + \frac{1}{3} = \frac{9}{15} = \frac{3}{5}$

37. i. Clearly, G be the centroid of $\triangle$BCD, therefore coordinates of G are

$\left(\frac{3+4+2}{3}, \frac{0+3+3}{3}, \frac{1+6+2}{3}\right)$ = (3, 2, 3)

ii. Since, A $\equiv$ (0, 1, 2) and G = (3, 2, 3)

$\therefore \overrightarrow{AG}$ = (3 - 0)$\hat{i}$ + (2 - 1)$\hat{j}$ + (3 - 2)$\hat{k}$ = 3$\hat{i}$ + $\hat{j}$ + $\hat{k}$

$\Rightarrow |\overrightarrow{AG}|^2$ = $3^2 + 1^2 + 1^2$ = 9 + 1 + 1 = 11

$\Rightarrow |\overrightarrow{AG}| = \sqrt{11}$

iii. Clearly, area of $\triangle$ABC = $\frac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

Here, $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3-0 & 0-1 & 1-2 \\ 4-0 & 3-1 & 6-2 \end{vmatrix}$

= $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -1 \\ 4 & 2 & 4 \end{vmatrix}$

= $\hat{i}$(-4 + 2) - $\hat{j}$(12 + 4) + $\hat{k}$(6 + 4) = -2$\hat{i}$ - 16$\hat{j}$ + 10$\hat{k}$

$\therefore |\overrightarrow{AB} \times \overrightarrow{AC}| = \sqrt{(-2)^2 + (-16)^2 + 10^2}$

= $\sqrt{4 + 256 + 100} = \sqrt{360} = 6\sqrt{10}$

Hence, area of $\triangle$ABC = $\frac{1}{2} \times 6\sqrt{10} = 3\sqrt{10}$ sq. units

OR

The length of the perpendicular from the vertex D on the opposite face

= |Projection of $\overrightarrow{AD}$ on $\overrightarrow{AB} \times \overrightarrow{AC}$|

= $\left|\frac{(2\hat{i}+2\hat{j})\cdot(-2\hat{i}-16\hat{j}+10\hat{k})}{\sqrt{(-2)^2+(-16)^2+10^2}}\right|$

= $\left|\frac{-4-32}{\sqrt{360}}\right| = \frac{36}{6\sqrt{10}} = \frac{6}{\sqrt{10}}$ units

38. i. Given, r cm is the radius and h cm is the height of required cylindrical can.

Given that, volume of cylinder = 3l = 3000 cm^3 ($\because$ 1l = 1000 cm^3)

$\Rightarrow \pi r^2 h$ = 3000 $\Rightarrow$ h = $\frac{3000}{\pi r^2}$

Now, the surface area, as a function of r is given by

S(r) = $2\pi r^2 + 2\pi rh = 2\pi r^2 + 2\pi r\left(\frac{3000}{\pi r^2}\right)$

= $2\pi r^2 + \frac{6000}{r}$

ii. Now, S(r) = $2\pi r^2 + \frac{6000}{r}$

$\Rightarrow$ S'(r) = $4\pi r - \frac{6000}{r^2}$

To find critical points, put S'(r) = 0

$\Rightarrow \frac{4\pi r^3 - 6000}{r^2}$ = 0

$\Rightarrow$ $r^3 = \frac{6000}{4\pi} \Rightarrow r = \left(\frac{1500}{\pi}\right)^{1/3}$

Also, $S''(r)|_{r} = \sqrt[3]{\frac{1500}{\pi}} = 4\pi + \frac{12000\times\pi}{1500}$

= $4\pi + 8\pi$ = 12π > 0

Thus, the critical point is the point of minima.

iii. The cost of material for the tin can is minimized when r = $\sqrt[3]{\frac{1500}{\pi}}$ cm and the height is $\frac{3000}{\pi\left(\sqrt[3]{\frac{1500}{\pi}}\right)^2} = 2\sqrt[3]{\frac{1500}{\pi}}$ cm.

OR

We have, minimum surface area = $\frac{2\pi r^3 + 6000}{r}$

= $\frac{2\pi \cdot \frac{1500}{\pi} + 6000}{\sqrt[3]{\frac{1500}{\pi}}} = \frac{9000}{7.8}$ = 1153.84 cm^2

Cost of 1 m^2 material = ₹100

$\therefore$ Cost of 1 cm^2 material = ₹ $\frac{1}{100}$

$\therefore$ Minimum cost = ₹ $\frac{1153.84}{100}$ = ₹11.538